

Määratud integraal

Ülesanded koos lahendustega

I Leida järgmised integraalid

Ülesanne 1

$$\int_1^2 (3x^2 - x + 1) dx = \left[3 \cdot \frac{x^3}{3} - \frac{x^2}{2} + x \right]_1^2 = \left(x^3 - \frac{x^2}{2} + x \right) \Big|_1^2 = (8 - 2 + 2) - \left(1 - \frac{1}{2} + 1 \right) = 8 - 1\frac{1}{2} = 6\frac{1}{2}$$

Ülesanne 2

$$\int_0^{\frac{\pi}{2}} \cos x dx = \sin x \Big|_0^{\frac{\pi}{2}} = \sin \frac{\pi}{2} - \sin 0 = 1 - 0 = 1$$

Ülesanne 3

$$\int_{-2}^3 (x^2 - 2x + 3) dx = \left(\frac{x^3}{3} - 2 \frac{x^2}{2} + 3x \right) \Big|_{-2}^3 = \frac{3^3}{3} - 3^2 + 3 \cdot 3 - \left(\frac{(-2)^3}{3} - (-2)^2 + 3 \cdot (-2) \right) = 9 - \left(-\frac{8}{3} - 10 \right) = 9 + \frac{8}{3} + 10 = \frac{19 \cdot 3 + 8}{3} = \frac{65}{3} = 21\frac{2}{3}$$

Ülesanne 4

$$S = - \int_{-2}^2 (x^2 - 4) dx = - \left(\frac{x^3}{3} - 4x \right) \Big|_{-2}^2 = - \left(\frac{2^3}{3} - 4 \cdot 2 - \left(\frac{(-2)^3}{3} - 4 \cdot (-2) \right) \right) = - \left(\frac{8}{3} - 8 + \frac{8}{3} - 8 \right) = - \frac{16}{3} + 16 = \frac{16}{3} = 10\frac{2}{3}$$

Ülesanne 5

$$\int_1^3 \left(4 - x - \frac{3}{x} \right) dx = \left(4x - \frac{x^2}{2} - 3 \ln |x| \right) \Big|_1^3 = 12 - \frac{9}{2} - 3 \ln 3 - \left(4 - \frac{1}{2} - 0 \right) = \frac{15}{2} - 3 \ln 3 - \frac{7}{2} = 4 - 3 \ln 3$$

II Leida järgmised integraalid

1. $\int_0^{\frac{\pi}{4}} \cos x dx = \sin x \Big|_0^{\frac{\pi}{4}} = \sin \frac{\pi}{4} - \sin 0 = \frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{2}}{2}$

2. $\int_2^4 x dx = \frac{x^2}{2} \Big|_2^4 = \frac{4^2}{2} - \frac{2^2}{2} = 8 - 2 = 6$

3. $\int_{-1}^2 (6 + 4x) dx = (6x + 2x^2) \Big|_{-1}^2 = (6 \cdot 2 + 2 \cdot 2^2) - (6 \cdot (-1) + 2 \cdot (-1)^2) = 12 + 8 + 6 - 2 = 24$

4. $\int_2^0 \frac{2t + 3t^2}{3t} dt = \int_2^0 \left(\frac{2}{3} + t \right) dt = \left(\frac{2}{3}t + \frac{t^2}{2} \right) \Big|_2^0 = (0 + 0) - \left(\frac{4}{3} + 2 \right) = -3\frac{1}{3}$

8.
$$\int_0^1 (\sqrt[3]{t} - \sqrt{t}) dt = \int_0^1 (t^{1/3} - t^{1/2}) dt = \left(\frac{t^{1/3+1}}{1/3+1} - \frac{t^{1/2+1}}{1/2+1} \right) \Big|_0^1 = \left(\frac{3t^{4/3}}{4} - \frac{2t^{3/2}}{3} \right) \Big|_0^1 = \left(\frac{3}{4} - \frac{2}{3} \right) - 0 = \frac{1}{12}.$$