

# Lineaarvõrrandisüsteemid (lüh. LVS)

## I Lahenda LVS maatriksmeetodiga

1. 
$$\begin{cases} 3x - 4y = -27 \\ -6x - y = -9 \end{cases}$$

2. 
$$\begin{cases} 2x + y = 6 \\ 12x + 5y = 44 \end{cases}$$

3. 
$$\begin{cases} -4k + 2p = 14 \\ 10k - p = -3 \end{cases}$$

4. 
$$\begin{cases} x_1 + x_2 - 2x_3 = -4 \\ 2x_1 - 4x_2 - 3x_3 = -11 \\ 4x_1 - 2x_2 + 3x_3 = 11 \end{cases}$$

5. 
$$\begin{cases} 2x_1 - 2x_3 = -5 \\ x_1 + 4x_2 - 2x_3 = 9 \\ x_2 - 0,25x_3 = -3 \end{cases}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2, \\ \dots \dots \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n. \end{cases}$$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}$$

$$X = A^{-1}B$$

Vastused.

1.  $x = \frac{1}{3}; y = 7$

2.  $x = 7; y = -8$

3.  $k = 0,5; p = 8$

4.  $x_1 = 1; x_2 = 1; x_3 = 3$

5. Ei ole võimalik lahendada maatriksmeetodiga

## II Lahendada LVS Crameri valemitega

6. 
$$\begin{cases} -4x + 3y = 38 \\ 2x - y = -16 \end{cases}$$

7. 
$$\begin{cases} -3u + 5v = -51 \\ 6u - v = 12 \end{cases}$$

8. 
$$\begin{cases} x + 2y - z = -3 \\ 2x - y + z = 8 \\ 3x + 3y - 2z = -3 \end{cases}$$

9. 
$$\begin{cases} x + 4y - 3z = 2 \\ x - 5y - 3z = 1 \\ -x + 6y + 4z = -1 \end{cases}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2, \\ \dots \dots \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n. \end{cases}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \quad |A| = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

$$x_i = \frac{\Delta x_i}{|A|}, \text{ kus } \Delta x_i = \begin{vmatrix} a_{11} & \dots & a_{1,i-1} & b_1 & a_{1,i+1} & \dots & a_{1n} \\ a_{12} & \dots & a_{2,i-1} & b_2 & a_{2,i+1} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & \dots & a_{n,i-1} & b_n & a_{n,i+1} & \dots & a_{nn} \end{vmatrix}$$

10. 
$$\begin{cases} x_1 + x_2 = 2 \\ x_2 + x_3 = 3 \\ x_3 + x_1 = 4 \end{cases}$$

11. 
$$\begin{cases} 4x + y + z = 1 \\ x - 2y - z = -5 \\ 6x + y + z = 1 \end{cases}$$

12. 
$$\begin{cases} -x + 2y - 3z = 3 \\ 2x - y = 1 \\ -x + z = 1 \end{cases}$$

13. 
$$\begin{cases} x + y - 3z = 1 \\ 2x - y = -1 \\ -x + 2y - 3z = 2 \end{cases}$$

Vastused.

6.  $x = -5; y = 6$

8.  $x = 2; y = -1; z = 3$

10.  $x_1 = 1,5; x_2 = 0,5; x_3 = 2,5$

12. Lahend puudub

7.  $u = \frac{1}{3}; v = -10$

9.  $x = \frac{11}{9}; y = \frac{1}{9}; z = \frac{1}{9}$

11.  $x = 0; y = 4; z = -3$

13. Lõpmata palju lahendeid

Lisaharjutamiseks (lahendada maatriksmeetodiga või Crameri valemitega)

$$2.1. \begin{cases} 2x + y = 6 \\ 12x + 5y = 44 \end{cases}$$

$$2.2. \begin{cases} 3,3x + 2,5y = 3 \\ 3x + 2,3y = 6 \end{cases}$$

$$2.3. \begin{cases} 3x + 2y = 7 \\ 4x - 5y = 40 \end{cases}$$

$$2.4. \begin{cases} ax - 3y = 1 \\ ax - 2y = 2 \end{cases}$$

$$2.5. \begin{cases} x + 4y - 3z = 2 \\ x - 5y - 3z = 1 \\ -x + 6y + 4z = -1 \end{cases}$$

$$2.6. \begin{cases} 3x - y + 2z = -2 \\ x + 4y - 3z = -5 \\ 2x + 2y + z = 4 \end{cases}$$

$$2.7. \begin{cases} 5x + 3y + z = 0 \\ 3x + y - 3z = 1 \\ 6x + 3y + z = -5 \end{cases}$$

$$2.8. \begin{cases} 4x + y + z = 1 \\ x - 2y - z = -5 \\ 6x + y + z = 1 \end{cases}$$

$$2.9. \begin{cases} x + y - 2z = -4 \\ 2x - 4y - 3z = -11 \\ 4x - 2y + 3z = 11 \end{cases}$$

$$2.10. \begin{cases} 4x + 2y + z = 31 \\ 2x + y + 5z = 29 \\ x - y + 3z = 10 \end{cases}$$

$$2.11. \begin{cases} 2x_1 + x_2 + x_3 = -1 \\ 2x_1 - x_2 + 2x_3 = -4 \\ 4x_1 + x_2 + 4x_3 = -2 \end{cases}$$

$$2.12. \begin{cases} x_1 + 2x_2 + 3x_3 = 5 \\ x_1 + 3x_2 + 2x_3 = 1 \\ 3x_1 + x_2 + 2x_3 = 11 \end{cases}$$

$$2.13. \begin{cases} 2x - y + z = 2 \\ 3x + 2y + 2z = -2 \\ x - 2y + z = 1 \end{cases}$$

$$2.14. \begin{cases} x - 2y - 3z = 5 \\ 2x - y - z = 1 \\ x + 3y + 4z = 6 \end{cases}$$

$$2.15. \begin{cases} -x + 2y + z = 1 \\ 4x + y - 2z = 5 \\ 3x - y + 2z = 2 \end{cases}$$

$$2.16. \begin{cases} 2x - 5y + 3z = 5 \\ 3x - 7y + 3z = -1 \\ 5x - 9y + 6z = 7 \end{cases}$$

$$2.17. \begin{cases} -3x + 2y + z = -5 \\ 3x - 3y + 2z = 1 \\ 2x + y + 2z = 2 \end{cases}$$

$$2.18. \begin{cases} x_1 - 2x_2 + 2x_3 - 4x_4 = 2 \\ -2x_1 + 3x_2 - 4x_3 + 6x_4 = -1 \\ 3x_1 - 6x_2 + 5x_3 - 10x_4 = -2 \\ -6x_1 + 9x_2 - 10x_3 + 15x_4 = 3 \end{cases}$$

Vastused:

**2.1.**  $x = 76; y = -8$ . **2.2.**  $x = -906; y = 120$ . **2.3.**  $x = 5; y = -4$ .

**2.4.**  $x = \frac{4}{a}; y = 1$ . **2.5.**  $x = \frac{11}{9}; y = \frac{1}{9}; z = -\frac{1}{9}$ . **2.6.**  $x = -\frac{13}{5}; y = 0$ ;

$z = \frac{104}{25}$ . **2.7.**  $x = -5; y = 9,1; z = -2;3$  **2.8.**  $x = 0; y = 4; z = -3$ .

**2.9.**  $x = 1; y = 1; z = 3$  **2.10.**  $x = 5; y = 4; z = 3$ . **2.11.**  $x_1 = -2$ ;

$x_2 = 2; x_3 = 1$ . **2.12.**  $x_1 = 3; x_2 = -2; x_3 = 2$ . **2.13.**  $x = 2; y = -1; z = -3$ .

**2.14.**  $x = 3; y = 17; z = -12$ . **2.15.**  $x = 1; y = 1; z = 0$ . **2.16.**  $x = -4; y = 1; z = 6$ .

**2.17.**  $x = \frac{51}{29}; y = \frac{20}{29}; z = -\frac{32}{29}$ . **2.18.**  $x_1 = 20; x_2 = 17; x_3 = -12; x_4 = -10$ .