Funktsiooni tuletiste leidmine
(tuleiste tabel + diferentseerimisreeglid nr 2 ja 3)
ehk
korrutise ja jagatise tuletis

Näide 1

$$\left(x^3\cdot e^x\right)'=\left(x^3\right)'\cdot e^x+x^3\cdot\left(e^x\right)'=3\cdot x^2\cdot e^x+x^3\cdot e^x=$$
$$=x^2e^x\left(3+x\right)$$

Näide 2

$$\left(\left(x+2\right)\cdot\left(x^2-x+1\right)\right)'=\left(x+2\right)'\cdot\left(x^2-x+1\right)+\left(x+2\right)\cdot\left(x^2-x+1\right)'=$$
$$=1\cdot\left(x^2-x+1\right)+\left(x+2\right)\cdot\left(2x-1\right)=$$
$$=x^2-x+1+x^2-x+4x-2=$$
$$=2x^2+2x-1$$

Näide 3

$$\left(\frac{x^2}{2+x}\right)'=\frac{\left(x^2\right)'\cdot\left(2+x\right)-x^2\cdot\left(2+x\right)'}{\left(2+x\right)^2}=$$
$$=\frac{2x\cdot\left(2+x\right)-x^2\cdot\left(0+1\right)}{\left(2+x\right)^2}=$$
$$=\frac{4x+2x^2-x^2}{\left(2+x\right)^2}=$$
$$=\frac{x^2+4x}{\left(2+x\right)^2}$$

Näide 4

$$\left(\frac{\ln x}{x^2}\right)'=\frac{\left(\ln x\right)'\cdot x^2-\ln x\cdot\left(x^2\right)'}{\left(x^2\right)^2}=$$
$$=\frac{\frac{1}{x}\cdot x^2-\ln x\cdot 2x}{x^4}=$$
$$=\frac{x-2x\ln x}{x^4}=$$
$$=\frac{x\left(1-2\ln x\right)}{x^4}=$$
$$=\frac{1-2\ln x}{x^3}$$

Näide 5

$$\left(\frac{\sin x}{\cos x}\right)'=\frac{\left(\sin x\right)'\cdot\cos x-\sin x\cdot\left(\cos x\right)'}{\left(\cos x\right)^2}=$$
$$=\frac{\cos x\cdot\cos x-\sin x\cdot\left(-\sin x\right)}{\cos^2 x}=$$
$$=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=$$
$$=\frac{1}{\cos^2 x}$$

Näide 6

$$f(x) = \frac{3x^2 + 5x}{x^3}$$

$$f'(x) = \frac{(6x+5) * x^3 - (3x^2 + 5x) * 3x^2}{x^6} = \frac{6x^4 + 5x^3 - 9x^4 - 15x^3}{x^6}$$

$$= \frac{-3x^4 - 10x^3}{x^6} = -\frac{3}{x^2} - \frac{10}{x^3}$$

Näide 7

$$y = \frac{x+2}{x-2}$$

$$f = x + 2$$

$$f' = 1$$

$$g = x - 2$$

$$g' = 1$$

$$g^2 = (x-2)^2$$

$$y' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$= \frac{1 \cdot (x-2) - (x+2) \cdot 1}{(x-2)^2}$$

$$= \frac{-4}{(x-2)^2}$$

Näide 8

$$y = \frac{x^2 + 3 \cdot x - 7}{x^2 + x^4 + 1}$$

$$f = x^2 + 3 \cdot x - 7$$

$$f' = 2 \cdot x + 3$$

$$g = x^2 + x^4 + 1$$

$$g' = 2 \cdot x + 4 \cdot x^3$$

$$g^2 = (x^2 + x^4 + 1)^2$$

$$y' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$= \frac{(2 \cdot x + 3) \cdot (x^2 + x^4 + 1) - (x^2 + 3 \cdot x - 7) \cdot (2 \cdot x + 4 \cdot x^3)}{(x^2 + x^4 + 1)^2}$$

Näide 9

Leida y', kui $y = (x^4 - x) \cdot (3\tan x - 1)$.

Lahendus. Kasutame funktsioonide korrutise tuletise võtmise reeglit

$$(u \cdot v)' = u' \cdot v + u \cdot v'.$$

Praegu $u = x^4 - x$ ja $v = 3\tan x - 1$ ning saame, et

$$y' = \left[(x^4 - x) \cdot (3\tan x - 1)\right]' = (x^4 - x)'(3\tan x - 1) + (x^4 - x)(3\tan x - 1)' =$$

$$= (4x^3 - 1)(3\tan x - 1) + (x^4 - x) \cdot \frac{3}{\cos^2 x}.$$

Näide 10

Leida y', kui $y = \frac{1 + e^x}{1 - e^x}$.

Lahendus. Kasutame funktsioonide jagatise tuletise võtmise reeglit

$$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}.$$

Praegu $u = 1 + e^x$ ja $v = 1 - e^x$ ning saame, et

$$y' = \left(\frac{1+e^x}{1-e^x}\right)' = \frac{\left(1+e^x\right)'\left(1-e^x\right)-\left(1+e^x\right)\left(1-e^x\right)'}{\left(1-e^x\right)^2} =$$

$$= \frac{e^x\left(1-e^x\right)-\left(1+e^x\right)\left(-e^x\right)}{\left(1-e^x\right)^2} = \frac{e^x - \cancel{e^{2x}} + e^x + \cancel{e^{2x}}}{\left(1-e^x\right)^2} = \frac{2e^x}{\left(1-e^x\right)^2}.$$

Näide 11

$$y = \frac{\cos x}{1+\sin x}.$$

$$y' = \frac{(\cos x)'(1+\sin x)-\cos x(1+\sin x)'}{(1+\sin x)^2} = \frac{-\sin x(1+\sin x)-\cos x\cos x}{(1+\sin x)^2} =$$

$$= \frac{-\sin x-\sin^2 x-\cos^2 x}{(1+\sin x)^2} = \frac{-(1+\sin x)}{(1+\sin x)^2} = -\frac{1}{1+\sin x}.$$

Näide 12

$$y = 4x\cdot\cos x+\tan x$$

$$y' = 4\cdot(x)'\cdot\cos x+4x\cdot(\cos x)'+(\tan x)' = 4\cos x-4x\sin x+\frac{1}{\cos^2 x}$$

Näide 13

$$f(x) = \frac{3x^2-2x-4}{2x-1}.$$

$$f'(x) = \frac{(3x^2-2x-4)'(2x-1)-(3x^2-2x-4)(2x-1)'}{(2x-1)^2} =$$

$$= \frac{(6x-2)(2x-1)-(3x^2-2x-4)\cdot 2}{(2x-1)^2} = \frac{6x^2-6x+10}{(2x-1)^2}.$$

Näide 14

$$f(x) = \frac{1+\cos x}{1-\sin x}.$$

$$f'(x) = \frac{(1+\cos x)'(1-\sin x)-(1+\cos x)(1-\sin x)'}{(1-\sin x)^2} =$$

$$= \frac{-\sin x\cdot(1-\sin x)-(1+\cos x)\cdot(-\cos x)}{(1-\sin x)^2} = \frac{-\sin x+\sin^2 x+\cos x+\cos^2 x}{(1-\sin x)^2} =$$

$$= \frac{1-\sin x+\cos x}{(1-\sin x)^2}.$$