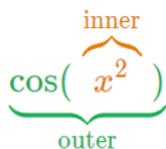


# 1. Quick review of composite functions

## 1.1. Composite Function

A function is **composite** if you can write it as  $f(g(x))$ .

In other words, it is a function within a function, or a function of a function. For example,  $\cos(x^2)$  is composite, because if we let  $f(x) = \cos(x)$  and  $g(x) = x^2$ , then  $\cos(x^2) = f(g(x))$ .



Function  $g$  is the function within  $f$ , so we call  $g$  the „inner“ function and  $f$  the „outer“ function.

### Common mistake: Not recognizing whether a function is composite or not

Usually, the only way to differentiate a composite function is using the chain rule. If we don't recognize that a function is composite and that the chain rule must be applied, we will not be able to differentiate correctly.

On the other hand, applying the chain rule on a function that isn't composite will also result in a wrong derivative.

Especially with transcendental functions (e.g., trigonometric and logarithmic functions), students often confuse compositions like  $\ln(\sin(x))$  products like  $\ln(x) \cdot \sin(x)$ .

On the other hand,  $\cos(x) \cdot x^2$  is **not** a composite function. It is the **product** of  $f(x) = \cos(x)$  and  $g(x) = x^2$ , but neither of the functions is within the other one.

**Example.** Have a look at the function  $f(x) = (x^2 + 1)^{17}$ . We can think of this function as being the result of combining two functions. Another way of representing this would be with a diagram like

$$x \xrightarrow{g} x^2 + 1 \xrightarrow{h} (x^2 + 1)^{17}.$$

We start off with  $x$ . The function  $g$  takes  $x$  to  $x^2 + 1$ , and the function  $h$  then takes  $x^2 + 1$  to  $(x^2 + 1)^{17}$ .

**Exercises.** Work out  $f(g(x))$  and  $g(f(x))$  for the following pairs of functions:

1.  $f(x) = 3x$ ,  $g(x) = 2x^2 + 1$
2.  $f(x) = e^{4x}$ ,  $g(x) = \sqrt{x}$
3.  $f(x) = \sin x$ ,  $g(x) = 1/x$

## 1.2. Order of composition

The order in which we compose functions makes a big difference to the result. You can see this if we change the order of the functions in the first example. We have taken  $f(x) = x^2$  and  $g(x) = x + 3$ . Then

$$f(g(x)) = f(x + 3) = (x + 3)^2 = x^2 + 6x + 9$$

and

$$g(f(x)) = g(x^2) = x^2 + 3.$$

In general  $g(f(x))$  is not equal to  $f(g(x))$ .

**Exercises.** Work out  $g(f(x))$  for the following pairs of functions and compare the results to those you obtained for Exercises 1-3.

4.  $f(x) = 3x$ ,  $g(x) = 2x^2 + 1$

5.  $f(x) = e^{4x}$ ,  $g(x) = \sqrt{x}$

6.  $f(x) = \sin x$ ,  $g(x) = 1/x$